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SOME PROPERTIES OF THE UNITARIES REPRESENTING MINIMAL INNER TORAL POLYNOMIALS

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An inner toral polynomial is a polynomial in two complex variables z and w such that its zero set is a subset of $\mathbb{D}^2 \cup \mathbb{T}^2 \cup \mathbb{E}^2$, where $\mathbb{D}$, $\mathbb{T}$, and $\mathbb{E}$ are the open unit disk, unit circle, and the exterior of the closed unit disk, respectively. A minimal inner toral polynomial is one that divides all the other inner toral polynomials with the same zero set as itself. In 2005, Jim Agler and John E. McCarthy proved that, for a given minimal inner toral polynomial $\mathfrak{p}(z,w)$ of degree n and m in z and w, respectively, there exists a unitary matrix, written in block form as $\begin{pmatrix} A & B \\ C & D \end{pmatrix}$, such that det $\begin{pmatrix} A - wI_m & zB \\ C & zD - I_n \end{pmatrix}$ is a constant multiple of $\mathfrak{p}(z,w)$. Moreover, the block matrix D in such unitaries has no unimodular eigenvalues. Greg Knese in 2010 gave an alternative proof to the same result. Following their work, we prove the following results on the block matrix D. If $\mathfrak{p}$ does not have the zw^m term, then the trace of the block matrix D is zero. Likewise, if $\mathfrak{p}$ does not have $z^n w^m$ term, then the determinant of the block matrix D is zero. Consequently, if $\mathfrak{p}$ does not have the $z^k w^m$ terms for all $k = 1,2,\dots,n$, then D has zero eigenvalues. Further, if $\mathfrak{p}$ does not have the $z^k w^m$ terms for all $k = 1,2,\dots,n-1$, then the determinant of D is not unimodular.

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