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ON A QUESTION OF CONSTRUCTING MÖBIUS TRANSFORMATIONS VIA SPHERES AND RIGID MOTIONS

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A Möbius transformation is a complex-valued function that maps points in the extended complex plane into itself either by translations, dilations, inversions, or rotations or even as a combination of the four mappings. Such a transformation can be constructed by a stereographic projection of the complex plane on to a sphere, followed by a rigid motion of the sphere, and a projection back onto the plane. In 2008, Arnold and Rogness created, and posted on YouTube, the video *Möbius Transformation Revealed* which became an instant hit. Their question, "*In how many different ways can the transformation be constructed using a sphere?*", which appeared in the follow-up paper was answered in 2012 by Siliciano, who showed that for any given Möbius transformation and an admissible sphere, there is exactly one rigid motion of the sphere with which the transformation can be constructed. The purpose of the present work is to work on a suggestion posted by Siliciano in characterizing rigid motions in constructing a specific Möbius transformation. Here our work shows that different admissible spheres under a unique Möbius transformation would require different rigid motions. In stereographic projections, the angle between lines on the surface of the sphere is equal to the angle between the projections of those lines and the circles on the surface of the sphere project as circles on the plane of projection. These are two most important existing results on stereographic projections and in the present work we have proved that under a unique Möbius transformation f, there exists different rigid motions $(T, \hat{T})$ for different admissible spheres $(S, \hat{S})$. Throughout the present work, Möbius transformations are written in the form $f = P_{T(S)} \circ T \circ P_S^{-1}$, where P_S is the stereographic motion before the rigid motion and $P_{T(S)}$ is the stereographic projection after the rigid motion. All these results can be combined and used in related applications, such as map making.

Keywords: Admissible sphere, Möbius transformation, Rigid motion, Stereographic projection