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FIRST ORDER KARUSH-KUHN-TUCKER CONDITIONS FOR QUADRATIC PROGRAMMING PROBLEMS WITH CONTINUOUS AND DISCRETE VARIABLES

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In classical optimization, method of Lagrange multiplier provides first order necessary conditions for optimization problems with equality constraints. Celebrated Karush-Kuhn-Tucker (KKT) conditions, published in 1951, generalize the Lagrange multiplier approach to Mathematical Programming problems with both equality and inequality constraints. In this research, a useful first order optimality conditions are provided for the following nonlinear quadratic programming model problem with continuous and discrete mixed bounded variables:

Model Problem (MP)

$$\min_{x\in\mathbb{R}^n} f_0(x) = \min_{x\in\mathbb{R}^n} \frac{1}{2}x^T A_0 x + a_0^T x + c_0$$

$$\text{subject to } f_j(x) = \frac{1}{2}x^T A_j x + a_j^T x + c_j \le 0, \quad \forall j \in \{1,2,\dots\dots\dots,m\}$$

$$x_i \in [u_i, v_i]\,, \qquad i \in I - \text{continuous variable,}$$

$$x_i \in \{u_i, v_i\}\,, \qquad i \in J - \text{discrete variable,}$$

where $I \cap J = \emptyset, I \cup J = \{1,2,\dots\dots..,n\}$. A_j=$\left(a_{st}^{(j)}\right)$ is an order n symmetric matrix, for all $j\epsilon\{0,1,..,m\}$. a_j=$\left(a_r^j\right)\epsilon\mathbb{R}^n$, $c_j\epsilon\mathbb{R}$ and u_i,$v_i\epsilon\mathbb{R}$ with $u_i < v_i$ for all $i\epsilon\{1,2,..,n\}$.

As MP admits discrete variables, available KKT type local necessary optimality conditions are not readily applicable to this problem**.** A new necessary optimality condition is derived as follows: If $\bar{x}\ \epsilon\ \widetilde{D}\ i$s a local minimizer of (MP), then

$X_i(\bar{x})\sum_{j=0}^{m}\lambda_j\left(A_j\bar{x} + a_j\right)_i \le 0\ ,\ \forall\ i\ \epsilon\ I$; where $\lambda_j\epsilon\mathbb{R}^+$; $\quad j = 1,2,\dots,m$ are the Lagrangian multipliers associated with $\bar{x}\epsilon\ \widetilde{D}$, λ_0=1 and $X_i(\bar{x}) = -1$ if $\bar{x}_i = u_i$, 1 if $\bar{x}_i = v_i$, $\nabla L(\bar{x},\lambda)_i$ if $\bar{x}_i\epsilon(u_i, v_i)$. The newly derived necessary condition is provided in terms of the data/coefficients of MP and easily verifiable without long computation. Further it can be useful to develop a numerical scheme to locate the local minimizers of MP.

Keywords: Karush-Kuhn-Tucker conditions, Mixed variables, Quadratic programming problem